PhD Macro

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Due: 23 January

Question 1. Let k^* be the golden rule capital/labour ratio and let $c^* = f(k^*) - (u+n)k$, where u is the depreciation rate and n is labour growth. Suppose that f has the same properties as in lecture 1. Analyse the non-linear first-order difference equation

$$\dot{k} = f(k^*) - (u+n)k - c$$

graphically when $c < c^*$.

Question 2. Solve

$$\min_y \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \text{ subject to } y(x_1) = y_1 \text{ and } y(x_2) = y_2$$

to demonstrate that the shortest distance between two points in a plane is a straight line..

Question 3. Find the Euler equation associated with optimizing

$$\int_{t_1}^{t_2} x(t) \sqrt{1 + \dot{x}(t)^2} dt.$$

Question 4. Solve the following opimization problem:

$$\min_x \int_0^1 \left(\dot{x}^2 + 12xt\right) dx \text{ subject to } x(0) = 0 \text{ and } x(1) = 1.$$

Question 5. Consider the following model of a small open economy with a fixed exchange rate. Variables are in logarithm form.

Aggregate demand y is given by

$$y = e - p + 10,$$

where e is the exchange rate expressed as the home currency price of foreign currency and p is the home currency price of the single good.

Equilibrium in the money market is given by

$$m - p = 20 - i,$$

where m is the money supply and i is the home nominal interest rate.

The foreign nominal interest rate is zero and uncovered interest parity holds; hence,

$$i = \dot{e}.$$

Sticky price adjustment implies that

$$\dot{p} = y - 20.$$

(a) Write the equilibrium as a matrix equation in e and p and find the relevant eigenvalues. Demonstrate that there exists a saddlepath.

(b) Solve for e and p as functions of time given the initial value p_0 and the requirement that the economy be on the saddle path.

(c) Solve for the saddle path. That is, find e in terms of p when the economy is on the saddle path.

(d) Suppose that initially the economy is at a steady state with $m = 10$. Suppose that m jumps to 20. This jump is completely unexpected and it is believed that m will remain at 20 forever. Find e and p as functions of time.

(e) Present a graphical analysis of the path from the old steady state to the new steady state and demonstrate that the jump in the money supply leads to exchange rate "overshooting".