

PhD Macro
Professor Anne Sibert
Due: 16 February

Question 1. Prove that if $f(x, y)$ is concave and twice differentiable on a rectangle S then for every (x, y) and (x^*, y^*) in S

$$f(x, y) - f(x^*, y^*) \leq f_x(x^*, y^*)(x - x^*) + f_y(x^*, y^*)(y - y^*).$$

Use Brock's model from class to answer Questions 2 and 3.

Question 2. Demonstrate that if $\mu > \beta$ the appropriate Inada conditions ensure that there is at least one fundamental solution m to:

$$\mu u_m(c, m) = (\mu - \beta)u_c(c, m).$$

Question 3. Assume that $\beta < \mu < 1$. We have shown that if $\{m_t\}$ satisfies

$$\beta u_c(c, m_{t+1})m_{t+1} = \mu[u_c(c, m_t) - u_m(c, m_t)]m_t, \quad t \in \mathbb{Z}_+, \quad (1)$$

then $\{m_t\}$ satisfies the transversality condition and is an equilibrium sequence of real balances. Demonstrate graphically that for every $u(c, m_t) = h(c) + v(m_t)$, equation (1) is unstable and deflationary bubbles are possible.