BSc Financial Economics: International Finance
Lecture 2, Part 2
Autumn 2013

ARBITRAGE AND PARITY CONDITIONS

I. TRIANGULAR ARBITRAGE

Triangular Arbitrage involves exploiting a price discrepancy between three currencies to make a profit. If it is cheaper (or more expensive) to buy a second currency by purchasing it with a first currency than it is to buy the second currency by using the first currency to purchase a third currency and using that third currency to purchase the second currency then an arbitrage opportunity exists. For there to be no arbitrage possibilities it must be that it costs the same amount of any currency 1 to purchase any currency 2 as it does to purchase any currency 3 and use that currency 3 to purchase currency 2.

Let e_2 be the price of currency 2 in terms of currency 1 and e_3 be the price of currency 3 in terms of currency 1 and e_{cross} be the cross rate: the price of currency 2 in terms of currency 3. Then it costs e_2 units of currency 1 to purchase currency 2 directly. Or, it costs e_{cross} units of currency 3 to purchase a unit of currency 2 and e_3e_{cross} units of currency 1 to purchase e_{cross} units of currency 3. Thus, the cross rate must satisfy $e_2 = e_3e_{cross}$.

Absence of Triangular Arbitrage Possibilities

Let e_2 be the currency-1 price of currency 2

Let e_3 be the currency-1 price of currency 3

Let e_{cross} be the currency-3 price of currency 2

$$e_{cross} = e_2 / e_3$$

Example 1. Suppose that the Norwegian krone is trading at 20.0000 kroner per dollar and that the Japanese yen is trading at 100.00 yen per dollar. Find the yen per krone exchange rate.

Yen / krone = (yen / dollar) x (dollar / krone)
= (yen / dollar) / (krone / dollar)
= 100.00 / 20.0000 = 5.0000

Alternatively, one can convert 1 krone into 1 / 20.0000 dollars. One can convert 1 / 20.0000 dollars into 100.00 x (1/20.0000) = 5.0000 yen.

Example 2. Suppose further that the pound is trading at 2.0000 dollars per pound. Find the yen per pound exchange rate.

Yen / pound = (yen / dollar) x (dollar / pound) = 100.00 x 2.0000 = 200.00

Alternatively, one can convert 1 pound into 2.0000 dollars. One can convert 2.0000 dollars into 100.00 x 2.0000 = 200.00 dollars.

Example 3. Suppose further that the euro is trading at 1.5000 dollars per euro. Find the euros per pound exchange rate.

Euros / pound = (euros / dollar) x (dollars / pound)
= (dollars / pound) / (dollars / euro)
= 2.0000 / 1.5000 = 1.3333

Alternatively, one can convert 1 pound into 2.0000 dollars. One can convert 2.0000 dollars into 2.0000 / 1.5000 = 1.3333 euros. (approximately)

Example 4. (Triangular Arbitrage with Spreads.) Suppose that a bank quotes the Norwegian krone at 20.0010 kroner per dollar with a spread of 000 – 020. Suppose that it quotes the pound at 2.0010 dollars per pound with a spread of 000 – 020. At what rate will the bank sell kroner for pounds? At what rate will the bank buy kroner for pounds?

The krone rates are 20.0000 – 20.0020 and the pound rates are 2.0000 – 2.0020.

The bank will sell kroner (buy dollars) at 20.0000 kroner per dollar
The bank will buy kroner (sell dollars) at 20.0020 kroner per dollar
The bank will sell pounds (buy dollars) at 2.0020 dollars per pound
The bank will buy pounds (sell dollars) at 2.0000 dollars per pound

So, if you sell the bank one pound, you get 2.0000 dollars. If you sell the bank 2.0000 dollars you get 40.0000 kroner. The bank will sell kroner at 40.0000 kroner per pound.

If you buy a pound from the bank it costs 2.0020 dollars. If you buy 2.0020 dollars from the bank it costs 20.0020 x 2.00020 = 40.0440 kroner (approximately). The bank will buy kroner at 40.0440 kroner per pound.

II. Covered Interest Arbitrage

Suppose that the pound is trading at e dollars per pound in the spot market and at f dollars per pound in the one-year forward market. Suppose that the interest rate on one-year dollar deposits is i and the interest rate on one-year pound deposits is i*.

An investor can invest one dollar and end up with 1 + i dollars in a year. Or, the investor can sell one dollar for pounds, invest the pounds and simultaneously enter into a one-year forward contract to buy the proceeds of the investment back in a year. Then, he ends up with (1 + i*) f/ e.

The accounts have the same risk and liquidity characteristics and there is no foreign exchange risk. Thus, the investor should be indifferent between the two investments. Otherwise, there is an arbitrage opportunity. Thus, $1 + i = f (1 + i^*) / e$.

Absence of Covered Interest Arbitrage Possibilities

Let e be the home-currency price of foreign currency in the spot market

Let f be the home-currency price of foreign currency in the one-year forward market

Let i be the interest rate on one-year home-currency accounts

Let i* be the interest rate on one-year foreign currency accounts

$$1 + i = (1 + i^*) f / e$$

If you are given any three of the variables i, e, f and i*, you can find the fourth.

Advice: do not memorise the formula. Derive it each time.

Example 5. Suppose that the krone trades at 20.0000 kroner per dollar in the spot market, the one-year dollar interest rate is one percent and one-year krone interest rate is two percent. Find the one-year forward kroner per dollar exchange rate.

You can invest one dollar and get 1.01 dollars at the end of the year. You can sell one dollar for kroner, invest the kroner and simultaneously enter into a one-year forward contract to buy the proceeds of the investment back in a year. In this case you end up with 20.0000 x (1.02) / f dollars.

Since 1.01 = 20.0000 x 1.02 / f, f = 20.1980.

III. UNCOVERED INTEREST PARITY

Uncovered interest parity is an assumption that it sometimes made in economic models. It will be a key assumption of the models used in this part of the course as it can simplify the analysis greatly.

Uncovered interest parity says that expected return in terms of home currency of converting the home currency into foreign currency, investing it in a foreign currency account and then converting the proceeds back into home currency should be equal to the return to investing the home currency in a home-currency account.

Let i_t be the home-currency interest rate between periods t and t+1 and let i_t^* be the foreign-currency interest rate between periods t and t+1. Further, let e_t be the spot exchange rate in period t, expressed as the home-currency price of the foreign currency and let e^e_{t+1} be market participants' expectation of the spot exchange rate in period t+1.[1]

Then uncovered interest parity implies that $1 + i_t = (1 + i_t^*)\, e^e_{t+1} / e_t$.

[1] The vagueness about what this expectation is or where it comes from is deliberate.

Uncovered Interest Parity

Let e_t be the home-currency price of foreign currency at time t

Let e^e_{t+1} be market participants' expectation of the home currency price of foreign currency at time t+1

Let i_t be the interest rate home-currency accounts between time t and time t+1

Let i_t^* be the interest rate on one-year foreign currency accounts between time t and time t+1

$$1 + i_t = (1 + i_t^*)\, e^e_{t+1} \;/\; e_t$$

Example 6. Suppose that the pound is trading at 2.0000 dollars per pound in the spot market and that market participants expect it to be trading at 2.0050 a year from now. The one-year dollar interest rate is two percent. What is the one-year pound interest rate?

We have 1.02 = (1 + i*) 2.0050 / 2.0000. This implies that i* = .0175. Thus, the one-year pound interest rate is 1.75 percent.

Uncovered interest parity implies that $(1 + i_t) / (1 + i_t^*) = e^e_{t+1} / e_t$. Subtracting one from each side yields $(i_t - i_t^*) / (1 + i_t^*) = (e^e_{t+1} - e_t)/ e_t$. If the foreign interest rate is small or the time between periods is small, then uncovered interest parity says that the difference between the home and foreign interest rates is approximately equal to market participants' expectation of the depreciation of the home currency. In a continuous time model this holds exactly and uncovered interest parity says that interest rate differential is equal to market participants' expectation of the proportional rate of depreciation of the home currency.

IV. The Law of One Price

The law of one price is an assumption or rule of thumb that says a good should cost the same amount of home currency whether you purchase it directly with the home currency or convert the home currency into foreign currency and purchase it with the foreign currency.

Let p be the price of the good in home currency, p* be the price of the good in foreign currency and e be the exchange rate, expressed as the home currency price of the foreign good.

Then it costs p units of the home currency to buy the good. Or, it costs p* units of the foreign currency. A unit of foreign currency costs e units of home currency, so p* units of foreign currency costs ep* units of home currency. Thus, the law of one price is p = ep*.

The Law of One Price

Let p be the home-currency price of the good

Let p* be the foreign-currency price of the good

Let e be the home-currency price of the foreign currency

$$e = ep^*$$

Example 7. If a cashmere sweater sells for 200 pounds at Harrods and the pound spot rate is two dollars per pound, then the same cashmere sweater should sell for $400 at Saks Fifth Avenue.

Trade restrictions or costs and imperfect competition frequently cause the law of one price to be violated.

V. PURCHASING POWER PARITY

We can extend the idea of the law of one price to the consumer price index. If P is the consumer price index in the home country and P* is the consumer price index in the foreign country, then the analogous relationship is $P = eP^*$. This is called *absolute purchasing power parity* and it is unlikely to hold, even if the law of one price holds for every good. This is because countries consume different baskets of goods.

Example 9. Suppose that that country F consumes two units of wine and one unit of beer and that country G consumes one unit of wine and two units of beer. Suppose the exchange rate is fixed at one and that initially the currency-F and currency-G prices of wine are one and the currency-F and currency-G prices of beer are equal to two. The price index in country F is then $2 \times 1 + 1 \times 2 = 4$. The price index in country G is $1 \times 1 + 2 \times 2 = 5$. Clearly absolute purchasing power parit does not hold.

Suppose we replace purchasing power parity with a weaker condition: $P = ceP^*$, where c is some strictly positive constant. Then differentiate both sides with respect to time to get $dP/dt = ce \times dP^*/dt + cP^* \, de/dt$. Divide the left side by P and the right side by ceP^* to get $(dP/dt)/P = (dP^*/dt)/P^* + (de/dt)/e$. Rewriting yields $\pi = \pi^* + d$, where $\pi = (dP/dt)/P$ is home inflation, $\pi^* = (dP^*/dt)/P^*$ is foreign inflation and $d = (de/dt)/dt$ is the rate of devaluation of the home currency. This relationship is called (relative) *purchasing power parity* (PPP).

Purchasing Power Parity

Let π be home-currency inflation

Let π* be foreign-currency inflation

Let e be the proportional rate of depreciation of the home currency

$$\pi = e\pi^*$$

Example 9. Consider the beer and wine scenario of example 8 again. Call the initial period zero and let country F be the home country. We found that $P_0 = 4$ and $P_0^* = 5$. Now suppose that the exchange rate remains fixed and that the price of beer doubles in both currencies. Then $P_1 = 2 \times 1 + 1 \times 4 = 6$ and $P_0^* = 1 \times 1 + 2 \times 4 = 9$. Thus, inflation in the home country is (6 – 4)/4 x 100 percent = 50 percent. Inflation in the foreign country is (9 – 5)/5 x 100 percent = 80 percent. Since the exchange rate does not change, purchasing power parity does not hold.

Purchasing power parity is has been the subject of a massive empirical literature. It seems a somewhat misguided literature. It makes sense to empirically test a hypothesis that one has shown to be true in a theoretical model; it makes less sense to test a hypothesis that one knows is unlikely to be true.

Purchasing power parity is more likely to hold in periods dominated by monetary shocks than in periods denominated by real shocks. It is some times used a quick rule of thumb to judge what the market exchange rate of a country that has had a fixed exchange rate for some time might be. It is also a convenient assumption in economic models.