# Lecture 4: The Overlapping-Generations Model

## 1 Introduction

The beautiful and mysterious overlapping-generations model is 53 years old (unless you read French and can go back to Allais (1947)) and it is has kept a generation of mathematically minded economists happily employed.

## 2 A Very Simple OLG Model

It was a dark and stormy night. (One does not usually get to use such lurid prose in an economics lecture!) A traveller arrives at an inn. The innkeeper says that he has an infinite number of beds, but unfortunately they are all taken. There is a solution, however. Each guest can simply move down a bed. Karl Shell (1971) tells this story from George Gamow's 1947 book *One Two Three ... Infinity* to suggest the intuition behind the famous welfare result in the overlapping generations model.

### 2.1 Karl Shell's economics of infinity

Suppose that time occurs in discrete intervals and stretches to infinity.[1],[2] The model begins with period one. Each period a single two-period-lived consumer is born. This produces the key demographic feature of the OLG model: that a consumer born at time-$t$ can trade with the consumer born at time $t-1$ only at time $t$ and with the consumer born at time $t+1$ only at time $t$.

There is a single good in the model: chocolate. Each consumer is endowed with one chocolate bar at birth. Chocolate cannot be stored across periods. Each consumer wishes only to consume as much chocolate as possible. Clearly the equilibrium allocation is that each consumer eats his own chocolate bar. Let $p_t$ be the price of a time-$t$ chocolate bar in terms of a time-0 chocolate bar. The allocation is supported by any price sequence that has the price of chocolate rising over time.

The allocation is not Pareto optimal however. This is an example of an economy where the First Theorem of Welfare Economics – which says that any competitive equilibrium supports a Pareto optimal allocation – fails. To see this failure, suppose that each young consumer were to give his chocolate bar to the currently living older consumer. Then instead of consumers eating chocolate when young, they eat it when old. They are indifferent between this new allocation and the old one as it does not change their total amount of

[1] This simple version of the model is appropriate for an undergraduate lecture and can be well received, especially if you bring a basket of chocolate bars along.

[2] This model is due to Shell, Karl, "The Economics of Infinity," *Journal of Political Economy* 79, 1971, 1002-1011.

chocolate. The old consumer living in period one (who was born and ate his endowment of chocolate before the model began) is made strictly better off, however. In the old allocation he had no chocolate when old; in the new allocation he does.

How can we attain this Pareto improvement in a decentralised way? Suppose that the old consumer living in period one received a chocolate bar that was different from any of the ones received by later generations: it had a wrapper. Suppose that he eats his chocolate bar and keeps the wrapper. In period one he announces to the young consumer born in period one that the wrapper is a claim on a chocolate bar. Suppose that it is indeed, common knowledge that the wrapper is a claim. Then he is indifferent about whether he has chocolate when young or when old and he trades his chocolate for the wrapper, which he presents to the young consumer of generation two. This process continues forever, effecting the Pareto-improving transfer.

## 2.2 The first theorem of welfare economics

Why did the First Theorem of Welfare Economics not hold? Here is the proof of the theorem:[3]

The strategy is to show that you cannot construct a feasible allocation that is liked at least as much as the equilibrium allocation by all consumers and liked strictly more by at least one consumer. Suppose we have such a candidate feasible allocation. One claims first that since the candidate allocation is merely a rearrangement of the endowment it must have a value (at equilibrium prices) that is no more than the value of the endowment. Second one notes that since satiation is ruled out, budget constraints are satisfied with equality and hence the value of the equilibrium allocation is equal to the value of the endowment. Third one notes that if a consumer is indifferent between his endowment and the candidate allocation, then his candidate allocation cannot cost less than the equilibrium allocation or he could have bought something he liked more than the equilibrium allocation. So, the value of his candidate allocation is at least as much as the value of his equilibrium allocation. Fourth, if a consumer strictly prefers his candidate allocation to his equilibrium allocation it must cost more or he would have bought it. So, putting points three and four together a Pareto improving allocation must cost strictly more than the equilibrium allocation. But this is not consistent with points one and two.

More formally, imagine a world with a finite number $I$ of consumers indexed by $i = 1, ..., I$ and a finite number $K$ of goods indexed by $k = 1, ..., K$. Each consumer $i$ is endowed with $e_k^i$ units of good $k$. Denote the competitive equilibrium price of good $k$ by $p_k$ and the resulting consumption of good $k$ by consumer $i$ by $c_k^i$. Suppose there exists an allocations $\left\{a_k^i\right\}$ that Pareto dominates the allocation supported by the competitive equilibrium.

[3] See David M. Kreps, *A Course in Microeconomic Theory*, Princeton, Princeton University Press, 1990, p. 199-200.

We have $\sum_{i=1}^{I} a_k^i \leq e_k$. This implies that

$$\sum_{k=1}^{K} p_k \sum_{i=1}^{I} a_k^i \leq \sum_{k=1}^{K} p_k e_k. \tag{1}$$

By Walras' Law and non-satiation

$$\sum_{i=1}^{I} \sum_{k=1}^{K} p_k c_k^i = \sum_{k=1}^{K} p_k e_k. \tag{2}$$

By assumption, consumer $i$ likes $a^i$ at least as much as $c^i$ and some consumer $j$ likes it more. Suppose consumer $i$ is indifferent between $a^i$ and $c^i$. If $\sum_{k=1}^{K} p_k c_k^i > \sum_{k=1}^{K} p_k a_k^i$ then consumer $i$ could buy something it prefers to $a^i$, and hence to $c^i$. Thus, $\sum_{k=1}^{K} p_k c_k^i \leq \sum_{k=1}^{K} p_k a_k^i$. Suppose that consumer $i$ strictly prefers $a^i$ to $c^i$, then $\sum_{k=1}^{K} p_k c_k^i < \sum_{k=1}^{K} p_k a_k^i$ or the consumer would have bought $a^i$. Adding up these inequalities yields

$$\sum_{i=1}^{I} \sum_{k=1}^{K} p_k c_k^i < \sum_{i=1}^{I} \sum_{k=1}^{K} p_k a_k^i. \tag{3}$$

Since $\sum_{i=1}^{I} \sum_{k=1}^{K} p_k a_k^i = \sum_{k=1}^{K} p_k \sum_{i=1}^{I} a_k^i$ inequalities (1) and (2) are not consistent with inequality (3) and we have a contradiction.

This proof does not work in the OLG model because there are an infinite number of consumers and an infinite number of goods (time-1 chocolate bars, time-2 chocolate bars, etc.) and, hence, it is not legitimate to swap the order of summation (Fubini's Theorem) as was done in the previous paragraph. The value of the endowments at the equilibrium price may not be finite.

# 3 Neil Wallace's Model of Fiat Money

This section considers Wallace's model of fiat money.[4]

## 3.1 The consumers' problem

Suppose that in each period $N_t$ two-period-lived consumers are born, where $N_{t+1}/N_t = n > 0$. There is a single good that can consumed, traded and stored. If a young consumer stores an amount $k$ of the good he receives $xk$ units when old. The consumer receives an endowment of $y$ when young and receives a transfer $T_{t+1}$ when old. Denote his consumption when young by $c_t^y$, his consumption when old by $c_{t+1}^o$, his savings of nominal balances by $M_t$, the amount of the good that he stores by $k_t$ and the time-$t$ money price of the

[4] Wallace, Neil, "The Overlapping Generations Model of Fiat Money," in Neil Wallace and John H. Kareken, eds., *Models of Monetary Economics*, Minneapolis, MN, Federal Reserve Bank of Minneapolis, 1980.

good by $P_t$. The optimisation problem of a young consumer born at $t > 0$ is

$$\max_{c_t^y, c_{t+1}^o} U\left(c_t^y, c_{t+1}^o\right) \text{ subject to}$$
$$\begin{aligned} c_t^y &= y - k_t - M_t/P_t \\ c_{t+1}^o &= xk_t + M_t/P_{t+1} + T_{t+1}, \end{aligned}$$

where it is assumed that the utility function has the usual nice properties and is, in addition, homothetic.[5]

A solution to the consumers problem with valued fiat money requires

$$\frac{U_1\left(y - M_t/P_t, M_t/P_{t+1} + T\right)}{U_2\left(y - M_t/P_t, M_t/P_{t+1} + T\right)} = P_t/P_{t+1} \text{ and } k_t = 0 \text{ if } P_t/P_{t+1} > x. \tag{4a}$$
$$\frac{U_1\left(y - k_t - M_t/P_t, xk_t + M_t/P_{t+1} + T_{t+1}\right)}{U_2\left(y - k_t - M_t/P_t, xk_t + M_t/P_{t+1} + T_{t+1}\right)} = P_t/P_{t+1} \text{ if } P_t/P_{t+1} = x. \tag{4b}$$
$$\frac{U_1\left(y - k_t, xk_t\right)}{U_2\left(y - k_t, xk_t\right)} = x \text{ if } P_t/P_{t+1} < x. \tag{4c}$$

Homotheticity implies that the marginal rate of substitution depends solely on the ratio of consumption when young to consumption when old. Thus we can define $v\left(c_t^y/c_{t+1}^o\right) := U_1\left(c_t^y, c_{t+1}^o\right)/U_2\left(c_t^y, c_{t+1}^o\right)$. The function $v$ is strictly decreasing. Hence we can define $h$ as the inverse function of $v$. We can thus rewrite equations (4a) and (4b) as

$$\frac{y - M_t/P_t}{M_t/P_{t+1} + T} = h\left(P_t/P_{t+1}\right) \text{ and } k_t = 0 \text{ if } P_t/P_{t+1} > x. \tag{5a}$$
$$\frac{y - k_t - M_t/P_t}{x\left(k_t + M_t/P_t\right) + T_{t+1}} = h\left(P_t/P_{t+1}\right) \text{ if } P_t/P_{t+1} = x. \tag{5b}$$

## 3.2 Market Clearing

Money market clearing requires

$$N_t M_t = M_t^s. \tag{6}$$

Substituting equation (6) into equation (5) yields

$$\frac{y - \frac{M_t^s}{N_t P_t}}{\frac{M_t^s}{N_t P_{t+1}} + T} = h\left(\frac{P_t}{P_{t+1}}\right) \text{ and } k_t = 0 \text{ if } P_t/P_{t+1} > x. \tag{7a}$$
$$\frac{y - k_t - \frac{M_t^s}{N_t P_t}}{xk_t + \frac{M_t^s}{N_t P_{t+1}} + T_{t+1}} = h\left(\frac{P_t}{P_{t+1}}\right) \text{ if } P_t/P_{t+1} = x. \tag{7b}$$

[5] A homothetic utility function is utility function that can be expressed as a strictly monotonic function of a homogeneous of degree one utility function. It has the property that the marginal rates of substitution are homogeneous of degree one. That is, the slope of the indifference curves is constant along any ray through the origin.

The government increases the money supply according to $M_{t+1}^s/M_t^s = z > 0$. It transfers the increase to the old so that

$$T_{t+1} = \frac{M_{t+1}^s - M_t^s}{N_t P_{t+1}} = \frac{(z-1) M_t^s}{N_t P_{t+1}}. \tag{8}$$

Substituting equation (8) into equation (7) yields

$$\frac{y - \frac{M_t^s}{N_t P_t}}{\frac{n M_{t+1}^s}{N_{t+1} P_{t+1}}} = h\left(\frac{P_t}{P_{t+1}}\right) \text{ and } k_t = 0 \text{ if } P_t/P_{t+1} > x. \tag{9a}$$

$$\frac{y - k_t - \frac{M_t^s}{N_t P_t}}{x k_t + \frac{n M_{t+1}^s}{N_{t+1} P_{t+1}}} = h\left(\frac{P_t}{P_{t+1}}\right) \text{ if } P_t/P_{t+1} = x. \tag{9b}$$

Define $m_t := M_t/P_t$. Note that

$$\frac{P_t}{P_{t+1}} = \frac{P_t}{M_t}\frac{M_{t+1}}{P_{t+1}}\frac{N_{t+1}M_t^s}{N_t M_{t+1}^s} = \frac{n m_{t+1}}{z m_t}. \tag{10}$$

Substituting equation (10) into equation (9) and rewriting yields

$$\frac{y - m_t}{n m_{t+1}} = h\left(\frac{n m_{t+1}}{z m_t}\right) \text{ and } k_t = 0 \text{ if } P_t/P_{t+1} > x. \tag{11a}$$

$$\frac{y - k_t - m_t}{x k_t + n m_{t+1}} = h\left(\frac{n m_{t+1}}{z m_t}\right) \text{ if } P_t/P_{t+1} = x. \tag{11b}$$

### 3.3 Existence and Optimality

**Proposition.** A necessary and sufficient condition for the existence of at least one monetary equilibrium is $xz/n \leq 1$.

**Proof.** *Necessity.* Suppose there is a monetary equilibrium with $xz/n > 1$.Then $m_{t+1}/m_t \geq zx/n > 1$. This unbounded sequence cannot satisfy the resource constraint.

*Sufficiency.* Let $k = 0$ and $m_t = m$, then there is a monetary equilibrium if there is an $m^*$ such that $(y - m^*)/(nm^*) = h(n/z)$. The left-hand side of the inequality goes to infinity as $m^*$ falls to zero and to zero as $m^*$ goes to $y$; hence such an $m^*$ must exist.

In the borderline case where $n/z = z$ there exist a continuum of fundamental equilibria with both money and storage. To see this note that for every $m = \alpha m^*$, where $\alpha \in (0,1)$, there exists a $k \in (0, y)$ that solves (11b).

Consider an equilibrium $k^*$ where money has no value. By the budget constraints and equation (4c) this requires

$$\frac{y - k^*}{x k^*} = h(x), \ c_t^{y*} = y - k^*, \ c_{t+1}^{o*} = x k^*, \ t > 0.$$

This equilibrium must satisfy

$$\begin{aligned} N_t c_t^{y*} + N_{t-1} c_t^{o*} &= N_t y + N_{t-1} x k^* - N_t k^*, \ t > 0 \Rightarrow \\ n c_t^{y*} + c_t^{o*} &= n y + (x - n) k^*, \ t > 0. \end{aligned}$$

If $x \leq n$ we could instead have each young consumer consume $y - k^*$. Then a total of $N_t k^*$ could be transferred to the $N_{t-1}$ old consumers, with each receiving $nk^* \geq xk^*$. Thus, the old of generations $t > 0$ are at least as well off and the old consumer of generation 0 is strictly better off. So, we have the result that if the return on storage is less than or equal to the rate of population growth the allocation associated with the equilibrium is not Pareto optimal.

It can also be demonstrated that if $x > n$, then any equilibrium allocation is Pareto optimal. This involves demonstrating that it is not possible to make Pareto improving transfers from the young to the old in this case.

As we saw, the fundamental monetary equilibrium exists if and only if $xz \leq n$. If $z > 1$ then the associated allocation $(c^{y*}, c^{o*})$ is not Pareto optimal. To see this, if $k_t = 0$ for every $t$ then feasibility requires

$$\begin{aligned} N_t c_t^y + N_{t-1} c_t^o &= N_t y, \ t > 0 \Rightarrow \\ c_t^y + \frac{c_t^o}{n} &= y, \ t > 0. \end{aligned}$$

Let $(\hat{c}^y, \hat{c}^o)$ maximise utility subject to

$$c^y + \frac{c^o}{n} = y.$$

The allocation $(c^{y*}, c^{o*})$ maximises utility in the set

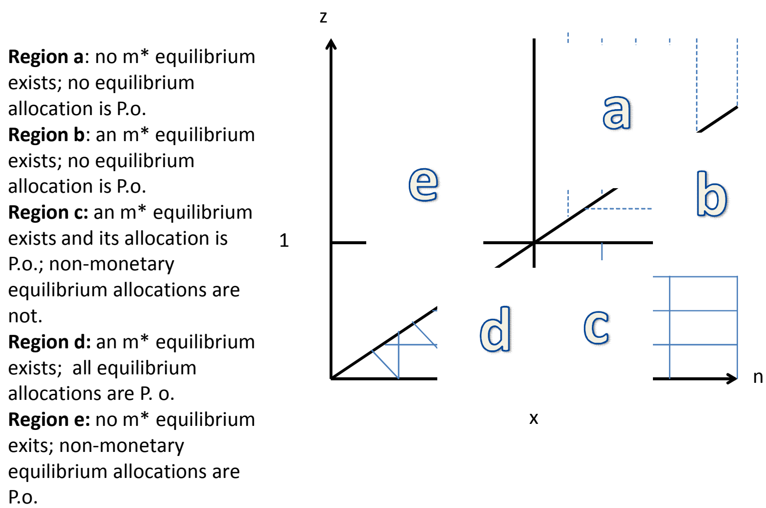
$$\begin{aligned} c^y &= y - k_t - M_t/P_t \\ c^o &= x k_t + M_t/P_{t+1} + T_{t+1} \Longleftrightarrow \end{aligned}$$

$$P_t c^y + P_{t+1} c^o = P_t y + P_{t+1} T_{t+1} \Leftrightarrow$$

$$c^y + \frac{P_{t+1} c^o}{P_t} = y + \frac{M_{t+1}^s - M_t^s}{N_t P_t} \Leftrightarrow$$

$$c^y + \frac{z c^o}{n} = y + (z - 1) m^*$$

As seen in the attached figure, the allocation $(c^{o*}, c^{y*})$ is at the intersection of the two budget constraints. The allocation $(\hat{c}^o, \hat{c}^y)$ must be to the southeast of $(c^{o*}, c^{y*})$ because it is preferred to $(c^{o*}, c^{y*})$ in the feasible




set. This allocation makes the old of generation born at time zero strictly better off.

If $z \leq 1$ then it can be shown that no Pareto-improving transfer from the young to the old exists and the allocation associated with the fundamental monetary equilibrium is Pareto optimal. The relationships between the parameters and the outcomes is shown in Figure 2.

# 4 The Overlapping Generations Model and Exchange Rate Indeterminacy

In this section I consider a slightly simplified version of Kareken and Wallace's exchange rate indeterminacy model.[6]

The model is as in the previous section except that there are two countries, each with its own fiat money, and we simplify things by assuming that $N_t = 1$ in each country and storage is not possible. The countries are referred to as the home and foreign countries. Initially, we will assume that there is portfolio autarky: consumers hold only their own countries money.

[6] Kareken, John H. and Neil Wallace, "On the Indeterminacy of Equilibrium Exchange Rates," *Quarterly Journal of Economics* 96, 1981, 207-22.

## 4.1 Portfolio Autarky

### 4.1.1 The consumers' problems

The consumer in the home country solves

$$\begin{aligned} & \max_{c_t^{hy}, c_{t+1}^{ho}} U\left(c_t^{hy}, c_{t+1}^{ho}\right) \text{ subject to} \\ c_t^{hy} &= y^h - M_t^{hh}/P_t \\ c_{t+1}^{ho} &= M_t^{hh}/P_{t+1} + T_{t+1}^h, \end{aligned}$$

where $M_t^{hh}$ is the home consumer's demand for home money, $P_t$ is the price of the single world good in terms of home money and $T_{t+1}^h$ is the transfer from the home government to old home consumers. Both the home and foreign consumer are assumed to have homothetic preferences.

The consumer in the foreign country solves

$$\begin{aligned} & \max_{c_t^{fy}, c_{t+1}^{fo}} U\left(c_t^{fy}, c_{t+1}^{fo}\right) \text{ subject to} \\ c_t^{fy} &= y^f - e_t M_t^{ff}/P_t \\ c_{t+1}^{fo} &= e_{t+1} M_t^{ff}/P_{t+1} + T_{t+1}^f, \end{aligned}$$

where $M_t^{ff}$ is the foreign consumer's demand for foreign money, $e_t$ is the home-currency price of the foreign good and $T_{t+1}^f$ is the transfer from the foreign government to the old foreign consumers.

First-order conditions for the optimisation problems are

$$\frac{y^h - M_t^{hh}/P_t}{M_t^{hh}/P_{t+1} + T_{t+1}^h} = h\left(\frac{P_t}{P_{t+1}}\right) \tag{12a}$$

$$\frac{y^f - e_t M_t^{ff}/P_t}{e_{t+1} M_t^{ff}/P_{t+1} + T_{t+1}^f} = h\left(\frac{e_{t+1} P_t}{e_t P_{t+1}}\right), \tag{12b}$$

where $h$ is the inverse of the marginal rate of substitution function as in the previous section.

### 4.1.2 Market clearing

Money market clearing requires $M_t^{ii} = M_t^{is}$, $i = h, f$. The governments increase the money supplies according to $M_{t+1}^{is}/M_t^{is} = z^i > 0$. They transfer the increase to their country's old so that

$$T_{t+1}^h = \frac{M_{t+1}^{hs} - M_t^{hs}}{P_{t+1}}, \; T_{t+1}^f = \frac{e_{t+1} M_{t+1}^{fs} - e_{t+1} M_t^{fs}}{P_{t+1}}. \tag{13}$$

Substitute equation (13) into equations (12) and market clearing into the result

$$\frac{y^h - \frac{M_t^{hs}}{P_t}}{\frac{M_{t+1}^{hs}}{P_{t+1}}} = h\left(\frac{P_t}{P_{t+1}}\right) \tag{14a}$$

$$\frac{y^f - \frac{e_t M_t^{fs}}{P_t}}{\frac{e_{t+1} M_{t+1}^{fs}}{P_{t+1}}} = h\left(\frac{e_{t+1} P_t}{e_t P_{t+1}}\right). \tag{14b}$$

Use the notation $m_t^h = M_t^{hs}/P_t$ and $m_t^f = e_t M_t^{fs}/P_t$ to rewrite (14a) and (14b)

$$\frac{y^i - m_t^i}{m_{t+1}^i} = h\left(\frac{m_{t+1}^i}{z^i m_t^i}\right), \; i = h, f. \tag{15}$$

A fundamental $m^{i*}$ satisfies

$$m^{i*} = \frac{y^i}{1 + h\left(1/z^i\right)}, \; i = h, f. \tag{16}$$

So,

$$\frac{m^{f*}}{m^{h*}} = \frac{e_t M_t^{fs}}{M_t^{hs}} = \frac{y^h}{y^f}\frac{1 + h\left(1/z^f\right)}{1 + h\left(1/z^h\right)} \Longrightarrow e_t = \frac{M_t^{hs}}{M_t^{fs}}\frac{y^h}{y^f}\frac{1 + h\left(1/z^f\right)}{1 + h\left(1/z^h\right)}. \tag{17}$$

## 4.2 Laissez Faire

In this section it is supposed that consumers can hold both currencies.

### 4.2.1 The consumers' problems

The consumer in the home country solves

$$\max_{c_t^{hy}, c_{t+1}^{ho}} U\left(c_t^{hy}, c_{t+1}^{ho}\right) \text{ subject to}$$

$$c_t^{hy} = y^h - M_t^{hh}/P_t - e_t M_t^{hf}/P_t$$

$$c_{t+1}^{ho} = M_t^{hh}/P_{t+1} + e_{t+1} M_t^{hf}/P_{t+1} + T_{t+1}^h,$$

where $M_t^{hf}$ is the home consumer's demand for foreign money. The consumer in the foreign country solves

$$\max_{c_t^{fy}, c_{t+1}^{fo}} U\left(c_t^{fy}, c_{t+1}^{fo}\right) \text{ subject to}$$

$$c_t^{fy} = y^f - M_t^{fh}/P_t - e_t M_t^{ff}/P_t$$

$$c_{t+1}^{fo} = e_{t+1} M_t^{fh}/P_{t+1} + e_{t+1} M_t^{ff}/P_{t+1} + T_{t+1}^f,$$

where $M_t^{fh}$ is the foreign consumer's demand for home money.

#### 4.2.2 No Arbitrage

In this perfect certainty model of fiat money no consumer is going to hold a currency that is expected to depreciate. Hence, if both monies are to have value we need the no arbitrage condition that $e_t = e$, for every $t$. In this case the optimisation problems can be rewritten as

$$\begin{aligned} & \max_{c_t^{iy}, c_{t+1}^{io}} U\left(c_t^{iy}, c_{t+1}^{io}\right) \text{ subject to} \\ c_t^{iy} &= y^i - s_t^i / P_t \\ c_{t+1}^{io} &= s_t^i / P_{t+1} + T_{t+1}^i, \text{ where} \\ s_t^i &= M_t^{ih} + e M_t^{fh}, \; i = h, f. \end{aligned}$$

With the no-arbitrage condition consumers care about total savings but not about the composition of their portfolios.

First-order conditions for the optimisation problems are

$$\frac{y^h - s_t^i / P_t}{s_t^i / P_{t+1} + T_{t+1}^i} = h\left(\frac{\beta P_t}{P_{t+1}}\right). \tag{18}$$

#### 4.2.3 Market clearing

Since we cannot disentangle home and foreign money demand, money market clearing requires $s_t^h + s_t^f = M_t^{hs} + e M_t^{fs}$, $i = h, f$. By (18) this gives us

$$y - \frac{M_t^{hs} + e M_t^{fs}}{P_t} = h\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{M_t^{hs} + e M_t^{fs}}{P_{t+1}} + T_{t+1}^h + T_{t+1}^h\right), \text{ where } y = y^h + y^f. \tag{19}$$

Substituting (13) into (19) yields

$$y - \frac{M_t^{hs} + e M_t^{fs}}{P_t} = h\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{M_{t+1}^{hs} + e M_{t+1}^{fs}}{P_{t+1}}\right). \tag{20}$$

The governments increase the money supplies according to $M_{t+1}^{is} / M_t^{is} = z^i > 0$. It simplifies the algebra without changing the fundamental result if we let $z^i = z$. Let

$$m_t = \frac{M_t^{hs} + e M_t^{fs}}{P_t}$$

and rewrite equation (20)

$$\frac{y - m_t}{m_{t+1}} = h\left(\frac{m_{t+1}}{z m_t^i}\right). \tag{21}$$

For every constant exchange rate there exists a fundamental equilibrium solving equation (21). Thus, the exchange rate cannot be determined. The intuition is that the exchange rate must be constant to ensure that both currencies are held. But, once that is true consumers are indifferent over different compositions of their portfolio and so instead of two market clearing conditions we have only one. This allows us to determine equilibrium real balances but not the exchange rate.

The real analogue to this scenario is not the determination of, say, the euro-pound exchange rate but the determination of the exchange rate between the 27.3 mm copper-nickel alloy heptagon shaped coins and the 22.5 mm nickel-brass alloy round coins that circulate in the United Kingdom. The government arbitrarily decided that the exchange rate was two of the heptagon shaped coins to one of the round ones, but it could have picked any exchange rate without needing to adjust the relative supplies to sustain it.

Note that adding uncertainty (say about endowments or money growth) does not make this problem of indeterminacy go away. Instead of imposing a constant exchange rate as a no-arbitrage condition, conjecture that the equilibrium has a constant exchange rate. Then consumers will not care how their portfolios are allocated and a market clearing condition is lost: the exchange rate is not determined.