Monetary Policy with Forward-Looking Agents

Anne Sibert

Spring 2014

1. Monetary Policy with Forward-Looking Agents

The model described here is based Barro and Gordon (1983).

It is assumed that social welfare is given by

$$W = -\pi^2 - \chi (n - n^o)^2 , \chi > 0. \tag{1}$$

where π is inflation, n is employment, and n^o is socially optimal employment. All three of these variables are in logarithm form. The interpretation of this is as follows. Optimal inflation is assumed to be zero. This is not important and it saves on notation. Society dislikes deviations from this socially optimal rate and welfare falls at an increasing rate as inflation rises above or falls below zero. Society also dislikes deviations from the socially optimal level of emplyment and welfare falls at an increasing rate as employment rises above or falls below this optimal level. The parameter χ tells us how society weights losses due to inflation deviations versus losses due to output deviations. The higher is χ, the greater the weight society places on employment deviations.

It is assumed that firms and workers enter into wage contracts. These contracts specify a fixed nominal wage W. *After* the wage is set, the price level is realised and firms decide how much labour to hire. Firms maximise their profits by setting the marginal product of labour equal to the real wage, W/P. Thus, the firms have a downward sloping labour demand curve. This is represented in Figure 1, where the horizontal axis represents employment N (in level, not logarithm form) and the vertical axis represents the real wage (in level, not logarithm form). The labor demand curve N^d equals the marginal product of labour.

Let N^*denote workers's and firms' most preferred level of employment, called the *natural rate.* Let P^e be the workers' and firms' expectation of the price level. This is where the model deviates from a Keynesian model and where forward looking expectations matter. In deciding what nominal wage to choose, the private sector forecasts the price level. It is initially assumed that there is no uncertainty in the model and that the private sector has perfect foresight. Thus, it will turn out that their expectation off the future price equals the actual price level. It is assumed that the workers' and firms choose the contractual nominal wage W such that if the price level turns out to be P^e, then the firms choose employment of n^*. Suppose that workers and firms are wrong and the price level turns out to be higher than expected, say $P^1 > P^e$. Then the firms choose employment of $n^1 < n^*$. If he price level turns out to be lower than expected, say $P^2 < P^e$. Then the firms choose employment of $N^2 < N^*$. In general, it is clear from the Figure that employment is increasing in P/P^e, with employment equal to N^* when P/P^e. We can write this idea in logarithm form as

$$n = n^* + \alpha (p - p^e) , \tag{2}$$

where α is normalised to one to simplify the notation. Equation (2) can be rewritten as

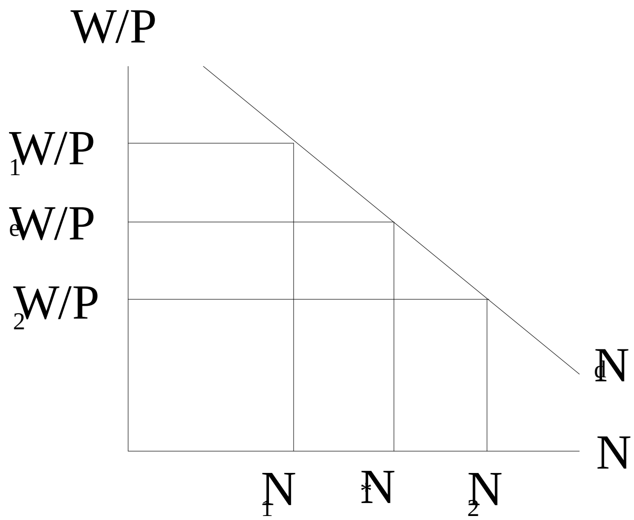

Figure 1: The Labour Market

$$n = n^* + (p - p_{-1}) - (p^e - p_{-1}) = n^* + \pi - \pi^e, \tag{3}$$

where p_{-1} is last period's price level (in logarithm form), $\pi = p - p_{-1}$is inflation and $\pi^e = p^e - p_{-1}$ is the private sector's expectation of inflation.

Equation (3) is an *expectations-augmented Phillips Curve*. It shows that there is a positive relationship between employment and *unexpected* inflation. The difference between this Phillips Curve and the conventional Philips curve is a result of the private sector anticipated the future actions of the government when it sets the contractual nominal wage.

Substituting equation (3) into equation (1) yields

$$W = -\pi^2 - \chi \left(n^* + \pi - \pi^e - n^o\right)^2. \tag{4}$$

It is assumed that $n^* < n^o$. This means that private sector's desired level of employment is less than the socially optimal rate. A reason for this might be distortionary taxes (such as income taxes) in labour markets. Let $d \equiv n^o - n^* > 0$ denote the deviation between the optimal and natural rates of employment. Use this notation to rewrite equation (4):

$$W = -\pi^2 - \chi \left(\pi - \pi^e - d\right)^2. \tag{5}$$

It is seen from equation (5) that because of the distortion in the labour market, the government wants to increase employment. It can do this, but at the cost of higher inflation.

Recall that the timing is that the private sector forms its expectation of inflation and incorporates it into its nominal wage contract. Then the central bank chooses monetary policy. This implies that when the central bank picks its policy, it takes expected inflation as given – that is, it treats it as a constant in its optimisation problem.

It is assumed that the central bank can perfectly control inflation. Thus, it chooses π to maximise equation (5). To find the first-order condition of the maximisation problem, differentiate W with respect to π and set the result equal to zero

$$dW/d\pi = -2\pi - 2\chi(\pi - \pi^e - d) = 0. \tag{6}$$

To verify that the solution to this is indeed a maximum use the second-order condition; differentiate (6) and verify the result is strictly negative.

$$d^2W/d\pi^2 = -2 - 2 < 0. \tag{7}$$

Solving equation (6) for inflation yields

$$\pi = \frac{\chi(\pi^e + d)}{1+\chi}. \tag{8}$$

Thus, we have that inflation is increasing in expected inflation. This is because the higher is expected inflation, the higher actual inflation has to be to create a given amount of unexpected inflation. Inflation is increasing in d because the greater the deviation between the optimal and the natural rates of employment, the more incentive the policy maker has to create unexpected inflation. Inflation is also rising in the parameter χ; this is because the greater the weight that the policy maker puts employment deviations relative to inflation deviations, the more he is willing to inflate.

The public knows the preferences of the policy maker; hence, the public can solve for inflation. Thus, we assume that the public's expectation of inflation equals actual inflation. Substituting $\pi^e = \pi$ into equation (8) yields

$$\pi = \frac{\chi(\pi + d)}{1+\chi} \Leftrightarrow \pi = \chi d. \tag{9}$$

Note from equation (3) that workers and firms get their most preferred level of employment, n^*. The outcome also has strictly positive inflation. Everyone would be better off if the policy maker could commit himself to inflation of zero. If zero inflation were expected, then workers and firms would still have employment of n^*. This is the optimal solution. But, the policy maker cannot attain the optimal solution. If zero inflation were expected, then by equation (8), the central bank would want to reneg and set

$$\pi = \frac{\chi d}{1+\chi}. \tag{10}$$

Anticipating this, the public will not expect zero inflation. Thus, we say that the optimal solution of zero inflation is not *time consistent.* This problem – that the government will choose not to follow its optimal policy is called the *time inconsistency problem.*

It appears that a solution is to appoint a *conservative* central banker – that is, a central banker who puts no weight on employment deviations relative to output. Such a central banker will always choose zero inflation.

2. Credibility vs. Flexibility

Suppose that the firms' production functions are subject to a shock that shifts the labour demand curve.[1] Assume that the timing of events is as follows:

[1] The model in this section is due to Rogoff (1985).

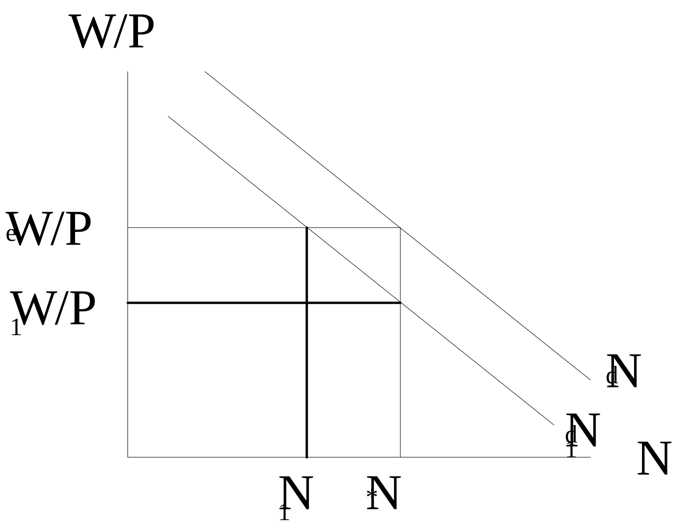

Figure 2: A Labour Demand Shock

1. Workers and firms enter into nominal wage contracts specifying W.
2. A stochastic mean-zero shock θ shifts the labour demand curve.
3. The central bank chooses inflation.

Consider Figure 1 again. Suppose that the workers and firms set $W = P^e$ and subsequently a shock shifts the labour demand curve in from N^d to N_1^d. This is depicted in Figure 2.

If the government sets inflation equal to what the private sector expected before it learned the realiszation of the shock, then employment will be $N_1 < N^*$. It is to the benefit of the private sector if the central bank inflates more than the private sector expected so that $P = P_1$ and $N = N^*$. Because the private sector forms its expectation of inflation and chooses W before the shock is realised and because the central bank chooses inflation after the shock is realised, the central bank has a *stabilisation* role.

Rewrite equation (5) as

$$W = -\pi^2 - \chi \left(\pi - \pi^e - d - \theta\right)^2 . \tag{11}$$

where θ is a shock that shifts the labour demand function in if it is positive and out if it is negative. Assume that θ has mean zero and variance σ^2.

The central bank chooses inflation after the shock has occurred, so it treats the shock as a constant and maximises equation (11). The first-order condition is

$$-2\pi - 2\chi \left(\pi - \pi^e - d - \theta\right) = 0, \tag{12}$$

Solving yields

$$\pi = \frac{\chi \left(\pi^e + d + \theta\right)}{1 + \chi}. \tag{13}$$

It is assumed that the private sector has rational expectations so that its expectation of inflation is the statistical expectation: $\pi^e = E\pi$. Then taking expectations of both sides of equation (13) yields

$$E\pi = E\left[\frac{\chi(E\pi + d + \theta)}{1+\chi}\right] = \frac{\chi(E\pi + d)}{1+\chi}. \tag{14}$$

Solving yields

$$E\pi = \chi d. \tag{15}$$

Substituting equation (15) into equation (13) yields

$$\pi = \frac{\chi(\chi d + d + \theta)}{1+\chi} = \chi d + \frac{\chi\theta}{1+\chi}. \tag{16}$$

Inflation is now equal to the *inflation bias* term χd that we found last time plus a stabilisation term $\chi\theta/(1+\chi)$.

Substituting equations (15) and (**??**) into equation (11) gives social welfare

$$W = -\left(\chi d + \frac{\chi\theta}{1+\chi}\right)^2 - \chi\left(\frac{\chi\theta}{1+\chi} - d - \theta\right)^2 = -\left(\chi d + \frac{\chi\theta}{1+\chi}\right)^2 - \chi\left(\frac{\theta}{1+\chi} + d\right)^2. \tag{17}$$

Taking the expected value yields

$$EW = -\left(\chi^2 + \chi\right)d^2 - \frac{\left(\chi^2+\chi\right)\sigma^2}{(1+\chi)^2} = -\chi(\chi+1)d^2 - \frac{\chi\sigma^2}{1+\chi}. \tag{18}$$

Suppose that instead the government appointed a conservative central banker or appointed an independent monetary policy committee and ordered it to follow a zero-inflation rule. Then $\pi = \pi^e$and welfare is

$$W = -\chi(d+\theta)^2. \tag{19}$$

Taking the expected value yields

$$EW = -\chi\left(d^2 + \sigma^2\right). \tag{20}$$

The conservative conservative central bank or rule is better than than the central banker who maximises welfare (this case is often called *discretion*) if

$$\begin{aligned} -\chi\left(d^2+\sigma^2\right) &> -\chi(\chi+1)d^2 - \frac{\chi\sigma^2}{1+\chi} \Leftrightarrow \chi\left(d^2+\sigma^2\right) < \chi(\chi+1)d^2 + \frac{\chi\sigma^2}{1+\chi} \qquad (21)\\ &\Leftrightarrow (1+\chi)d^2 + (1+\chi)\sigma^2 < (\chi+1)^2 d^2 + \sigma^2 \Leftrightarrow \sigma^2 < (1+\chi)d^2. \end{aligned}$$

This is sensible. A central banker who maximises social welfare tends to be bad because he produces an inflation bias and tends to be good because he stabilises. The conservative central banker does not produce an inflation bias or stabilise. Thus, if the variance of the labour-demand shock is sufficiently large, the stabilisation effect dominates and discretion is better than a rule. But, if the variance of the shock is sufficiently small, then stabilisation is not that important and the rule is better than discrestion. Society faces a tradeoff: If it appoints a conservative central banker or if it legislates a zero-inflation rule, it gains *credibility* (for low inflation), but it loses *flexibility* (in responding to shocks). A challenge is to come up with monetary institutions that confer credibility without sacrificing too much flexibility.

Barro, R. and D. Gordon (1983), "A Positive Theory of Monetary Policy in a Natural Rate Model, *Journal of Political Economy* 91, 589-610.

Rogoff, K. (1985), "The Optimal Degree of Commitment to an Intermediate Monetary Target," *Quarterly Journal of Economics* 100, 1169 - 1189.