I. Use the calculus of variations approach for parts (A) to (D)

A. Find and solve the Euler equation for

$$\int_0^1 (1+t)\dot{x}^2 dt \text{ s.t. } x(0)=0,\ x(1)=1.$$

B. Find and solve the Euler equation for

$$\int_1^2 \dot{x}(1+t^2\dot{x})dt \text{ s.t. } x(1)=2,\ x(2)=3/2.$$

C. Find the Euler equation for

$$\int_{t_0}^{t_1} x\sqrt{1+\dot{x}^2}\,dt.$$

D. Find and solve the Euler equations for

$$\int_0^1 x\dot{x}\,dt \text{ s.t. } x(0)=0,\ x(1)=1 \text{ and } \int_0^1 x\dot{x}t\,dt \text{ s.t. } x(0)=0,\ x(1)=1.$$

II. Consider the following variant of Dornbusch's model of a small open economy with sticky prices.

$$y = e - p + 10$$
$$m - p = 20 - i$$
$$i = \dot{e}$$
$$\dot{p} = y - 20,$$

where y is output, e is the exchange rate, p is the price level, m is the (constant) money supply and i is the nominal interest rate. (Variables are in logarithmic form.)

A. Solve for e and p, requiring that the economy is on the saddle path.

B. Suppose that the economy is at a steady state with m=10. There is an unexpected jump in the money supply to m = 20. Analyze graphically the path to the new steady state.

III. A representative agent in a closed economy faces the following problem.

$$\max \int_0^\infty e^{-\delta t}[\ln c + \ln(\bar{L} - L)]dt, \text{ s.t. } c + \dot{K} = 2\sqrt{KL} \text{ and } K(0) \text{ given},$$

where c is consumption, L is labor, K is the capital stock and δ and $\bar{L}$ are strictly positive constants.

A. Using control theory, find the Euler equations and the transversality condition.

B. Find c and L as functions of the costate variable and express the equilibrium two non-linear differential equations in K and the costate variable.

C. Linearize the system about the stationary point. Is the stationary point a saddle point? Explain.

D. Analyze the linearized equilibrium graphically.

IV. Consider a closed-economy overlapping-generations model with no storage, constant population growth and a constant money growth. The equilibrium satisfies

$$\frac{y-m_t}{m_{t+1}}=h\left(\frac{m_{t+1}}{m_t}\right),$$

where y is constant output, m_t is time-t real balances and the function h is the inverse of the marginal rate of substitution function.

A. What are the possible equilibria when the utility function is $\ln c_t^y+\beta\ln c_{t+1}^o$?

B. Can there be non-stationary equilibria when the utility function is $\sqrt{c_t^y}+\beta\sqrt{c_{t+1}^o}$? A graphical argument will suffice.